ADVANCED PROGRAMME MATHEMATICS

A. MEANS OF ASSESSMENT

Paper 1 (Core)	2 hours	[200]
Paper 2 (Elective)	1 hour	[100]

300 marks

The certification process for Advanced Programme Mathematics consists of a 3-hour external examination (paper 1 and paper 2) which is set by the Independent Examinations Board and quality assured by Umalusi.

B. REQUIREMENTS

To be read in conjunction with the: *Curriculum Statement Grades 10–12 (General) Advanced Programme Mathematics (2017).*

The Learning Outcomes of the Advanced Programme Mathematics National Curriculum Statement are divided into Core Learning Outcomes (LO 1: Calculus & LO 2 Algebra) and Elective Learning Outcomes (LO 3: Statistics, LO 4: Mathematical Modelling & LO 5: Matrices and Graph Theory). Learners will be examined on the Core Learning Outcomes (LO 1 & LO 2) and in addition, one of the Elective Outcomes (LO 3 or LO 4 or LO 5).

GRADE 12

EXAMINATION MARK ALLOCATION

Learning Outcome	Marks	Time
1	130–160	80–90 minutes
2	40–70	30–40 minutes
Elective	100	60 minutes
Total	300	3 hours

WEIGHTING ACCORDING TO TAXONOMY OF COGNITIVE LEVEL

Level		%
1	Knowledge	12–18
2	Routine procedures	37–43
3	Complex procedures	30–36
4	Problem-solving	7–13

DISTRIBUTION OF MARKS FOR CORE LEARNING OUTCOMES

Learning outcome	Topic	Mark distribution (±5)
1	Functions and limits	20
	Trigonometry	15
	Differentiation	35
	Integration	30
	Drawing functions	20
	Applications (max/min; rates of change; volume & area)	20
	Total	140
2	Real and complex roots	15
	Exponents and logarithms	15
	Absolute value	20
	Induction	10
	Total	60

DISTRIBUTION OF MARKS FOR ELECTIVE LEARNING OUTCOMES

Learning Outcome	Topic	Percentage
3	Probability fundamentals	15–30
	Probability functions and applications	50–60
	Inferential statistics	15–30
	Total	100
4	Graph theory	40–60
	Matrices	40–60
	Total	100
5	Financial models	40–60
	Recursive models	40–60
	Total	100

C. INTERPRETATION OF REQUIREMENTS

INFORMATION BOOKLET

Algebra

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad |x| = \begin{cases} x & ; \quad x \geq 0 \\ -x & ; \quad x < 0 \end{cases}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

$$z = a + bi \qquad z^* = a - bi$$

$$\ln A + \ln B = \ln (AB) \qquad \ln A - \ln B = \ln\left(\frac{A}{B}\right)$$

$$\ln A^n = n \ln A \qquad \log_a x = \frac{\log_b x}{\log_b a}$$

Calculus

$$\text{Area} = \lim_{n \to \infty} \left(\frac{b-a}{n}\right) \sum_{i=1}^{n} f\left(x_i\right) \qquad \int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\int f'\left(g(x)\right).g'(x)\,dx = f(g(x)) + c$$

$$\int f(x).g'(x)\,dx = f(x).g(x) - \int g(x).f'(x)\,dx + c$$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)} \qquad V = \pi \int_a^b y^2 dx$$

Function	Derivative
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x.\tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x.\cot x$
e^x	e^x
$\ln x$	$\frac{1}{x}$
$f(g(x))$	$f'(g(x)).g'(x)$
$f(x).g(x)$	$g(x).f'(x)+f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x).f'(x)-f(x).g'(x)}{[g(x)]^2}$

$$A=\frac{1}{2}r^2\theta \qquad s=r\theta$$

In $\triangle ABC$:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc.\cos A$$

$$Area=\frac{1}{2}ab.\sin C$$

$$\sin^2 A+\cos^2 A=1 \qquad 1+\tan^2 A=\sec^2 A \qquad 1+\cot^2 A=\text{cosec}^2 A$$

$$\sin(A\pm B)=\sin A.\cos B\pm\cos A\sin B \qquad \cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\begin{cases}\cos^2 A-\sin^2 A\\ 2\cos^2 A-1\\ 1-2\sin^2 A\end{cases}$$

$$\sin A.\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$$

$$\sin A.\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A.\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

Matrix Transformations

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

Finance & Modelling

$$F = P(1+in) \qquad F = P(1-in) \qquad F = P(1+i)^n \qquad F = P(1-i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1-(1+i)^{-n}}{i}\right] \qquad r_{eff} = \left(1+\frac{r}{k}\right)^k - 1$$

$$P_{n+1} = P_n + rP_n\left(1-\frac{P_n}{K}\right)$$

$$R_{n+1} = R_n + aR_n\left(1-\frac{R_n}{K}\right) - bR_nF_n \qquad F_{n+1} = F_n + f.bR_nF_n - cF_n$$

Statistics

$$P(A) = \frac{n(A)}{n(S)} \qquad P(B \mid A) = \frac{P(B \cap A)}{P(A)} \qquad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$${}^nP_r = \frac{n!}{(n-r)!} \qquad {}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!} \qquad P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(R = r) = \frac{\binom{p}{r}\binom{N-p}{n-r}}{\binom{N}{n}} \qquad E[X] = n \cdot p \qquad Var[X] = n \cdot p(1-p)$$

$$z = \frac{X-\mu}{\sigma} \qquad z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \qquad z = \frac{(\bar{x}-\bar{y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} \qquad p \pm z\sqrt{\frac{p(1-p)}{n}} \qquad E[X] = \sum x \cdot P(X = x)$$

$$Var[X] = E\left[X^2\right] - \left(E[X]\right)^2$$

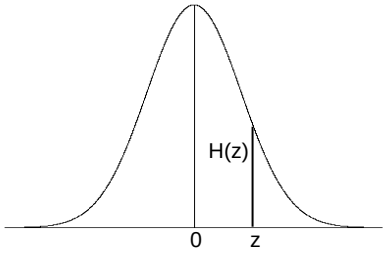
NORMAL DISTRIBUTION TABLE

Areas under the Normal Curve

$$H(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}x^2}\, dx$$

$H(-z) = H(z)$, $H(\infty) = \frac{1}{2}$

Entries in the table are values of H(z) for $z \geq 0$.



z	,00	,01	,02	,03	,04	,05	,06	,07	,08	,09
0,0	,0000	,0040	,0080	,0120	,0160	,0199	,0239	,0279	,0319	,0359
0,1	,0398	,0438	,0478	,0517	,0557	,0596	,0636	,0675	,0714	,0753
0,2	,0793	,0832	,0871	,0910	,0948	,0987	,1026	,1064	,1103	,1141
0,3	,1179	,1217	,1255	,1293	,1331	,1368	,1406	,1443	,1480	,1517
0,4	,1554	,1591	,1628	,1664	,1700	,1736	,1772	,1808	,1844	,1879
0,5	,1915	,1950	,1985	,2019	,2054	,2088	,2123	,2157	,2190	,2224
0,6	,2257	,2291	,2324	,2357	,2389	,2422	,2454	,2486	,2517	,2549
0,7	,2580	,2611	,2642	,2673	,2704	,2734	,2764	,2794	,2823	,2852
0,8	,2881	,2910	,2939	,2967	,2995	,3023	,3051	,3078	,3106	,3133
0,9	,3159	,3186	,3212	,3238	,3264	,3289	,3315	,3340	,3365	,3389
1,0	,3413	,3438	,3461	,3485	,3508	,3531	,3554	,3577	,3599	,3621
1,1	,3643	,3665	,3686	,3708	,3729	,3749	,3770	,3790	,3810	,3830
1,2	,3849	,3869	,3888	,3907	,3925	,3944	,3962	,3980	,3997	,4015
1,3	,4032	,4049	,4066	,4082	,4099	,4115	,4131	,4147	,4162	,4177
1,4	,4192	,4207	,4222	,4236	,4251	,4265	,4279	,4292	,4306	,4319
1,5	,4332	,4345	,4357	,4370	,4382	,4394	,4406	,4418	,4429	,4441
1,6	,4452	,4463	,4474	,4484	,4495	,4505	,4515	,4525	,4535	,4545
1,7	,4554	,4564	,4573	,4582	,4591	,4599	,4608	,4616	,4625	,4633
1,8	,4641	,4649	,4656	,4664	,4671	,4678	,4686	,4693	,4699	,4706
1,9	,4713	,4719	,4726	,4732	,4738	,4744	,4750	,4756	,4761	,4767
2,0	,4772	,4778	,4783	,4788	,4793	,4798	,4803	,4808	,4812	,4817
2,1	,4821	,4826	,4830	,4834	,4838	,4842	,4846	,4850	,4854	,4857
2,2	,4861	,4864	,4868	,4871	,4875	,4878	,4881	,4884	,4887	,4890
2,3	,48928	,48956	,48983	,49010	,49036	,49061	,49086	,49111	,49134	,49158
2,4	,49180	,49202	,49224	,49245	,49266	,49286	,49305	,49324	,49343	,49361
2,5	,49379	,49396	,49413	,49430	,49446	,49461	,49477	,49492	,49506	,49520
2,6	,49534	,49547	,49560	,49573	,49585	,49598	,49609	,49621	,49632	,49643
2,7	,49653	,49664	,49674	,49683	,49693	,49702	,49711	,49720	,49728	,49736
2,8	,49744	,49752	,49760	,49767	,49774	,49781	,49788	,49795	,49801	,49807
2,9	,49813	,49819	,49825	,49831	,49836	,49841	,49846	,49851	,49856	,49861
3,0	,49865	,49869	,49874	,49878	,49882	,49886	,49889	,49893	,49896	,49900
3,1	,49903	,49906	,49910	,49913	,49916	,49918	,49921	,49924	,49926	,49929
3,2	,49931	,49934	,49936	,49938	,49940	,49942	,49944	,49946	,49948	,49950
3,3	,49952	,49953	,49955	,49957	,49958	,49960	,49961	,49962	,49964	,49965
3,4	,49966	,49968	,49969	,49970	,49971	,49972	,49973	,49974	,49975	,49976
3,5	,49977									
3,6	,49984									
3,7	,49989									
3,8	,49993									
3,9	,49995									
4,0	,49997									